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A booklet for elementary school students to be used independently from and in addition to the regular classroom curriculum in mathematics is presented. Third-and fourth-grade students can gain additional insights into the concepts of scales, units of measure, age, dates, time, height, temperature, and money. Emphasis is put on the physical and practical application of recently learned arithmetic operations. The teacher should identify potential students, orient them to the area to be explored, and stimulate and maintain their interest in sustained independent effort.

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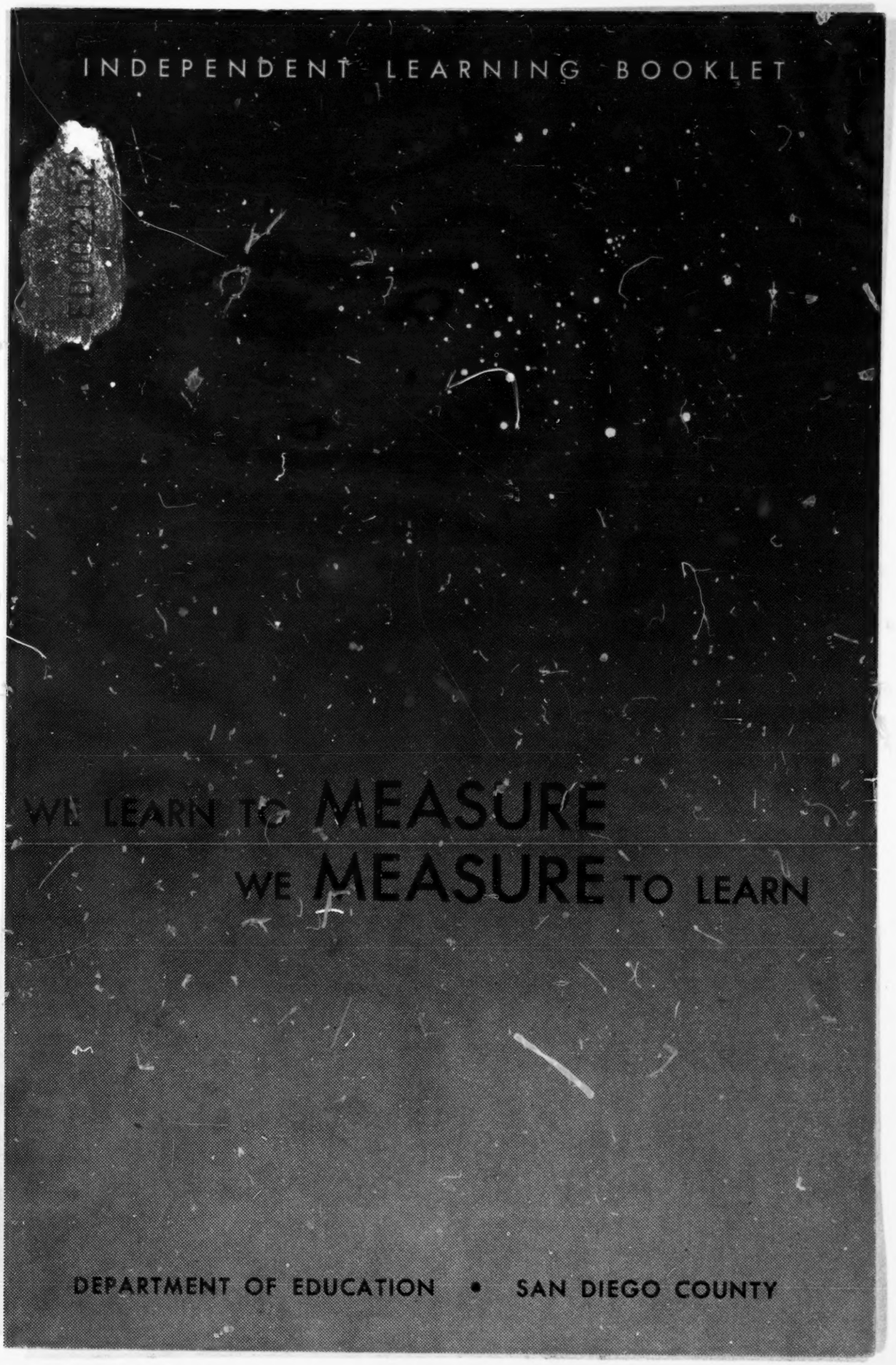

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TO THE TEACHER

This Independent Learning booklet is designed to help you open new avenues of learning for students who show a special interest in mathematics and a readiness to study on their own. Such students will be able to work directly from printed materials, using abstract ideas to follow a line of mathematical reasoning. Occasionally, of course, even the ablest of them will need your help to get over some of the rough spots, but your main concern in connection with the booklet will be to identify students who can benefit from using it, to orient them to the area to be explored, and to stimulate and maintain their interest in sustained independent effort.

Since the topic of the booklet trends outward from the regular curriculum, the challenge to do this independent work should arise from students' own searching in the field or from new interests that you have succeeded in awakening. Students will profit most if they elect voluntarily to do the work rather than receiving it as an assignment or as a means of earning extra credit.

The material is so organized that students may progress at their own best pace. They should be encouraged to move ahead in discovering for themselves meanings and approaches in mathematics beyond those usually taught in the classroom, but there should be no requirement to complete parts or all of the booklet, or to follow a set schedule.

While serious effort will be necessary, students should find the work enjoyable because of their interest in the subject. They should gain a real sense of satisfaction through working individually or with other students in independently achieving understanding, overcoming temporary difficulties, and evaluating their own progress. Your encouragement and occasional direct help will of course be needed.

This booklet obviously is not aimed at developing complete understanding of an area of mathematics. Its purpose is rather to provide an introduction, or an approach novel to the student, which will create new interests and the desire to go on from here to further study in other sources.

A brief teacher's guide has been prepared to assist you in using this booklet with your students. Copies of the guide are distributed to your district office by the Department of Education, San Diego County. Additional copies are always available.

INDEPENDENT LEARNING BOOKLET

We Learn to Measure
We Measure to Learn

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INTRODUCTION

This is the story of Bob and Betty—and how they learned to measure, and measured to learn. Read their story to see the kinds of questions they asked and the interesting answers they found.

Did YOU ever need to find out how big something was, how much it weighed, or how much it would hold? Have you ever wondered how we find the heights of tall trees and flagpoles? How we find how very hot or very cold some things are? Or how we developed our ways of telling time with clocks and calendars? Bob and Betty's story will help you answer these and other questions you may have asked.

You will also learn about some ways of measuring that are different than the ones we are used to. Your booklet explains how length was measured long ago, how people in foreign countries measure, how the calendar has changed. It shows how military men tell time differently than we do at home.

As you read the story you will find questions and problems after each explanation. Answer the questions and work the problems. They will help you to think clearly and to test your own learning. Check your thinking with the answers which come after each set of questions and problems. If your answers are right, go ahead. If you have missed some answers, look at your work to find your mistakes. You may have to go back to read the explanation again.

Your teacher will help you if you really get "stuck," but don't ask for help until you have studied the section of your booklet which explains the problem you are working on. Do this several times. You will feel more pleased if you work through the booklet on your own.

You will need paper and pencil to answer the questions and work the problems. Since other children will be using this booklet, please do not write in the booklet itself.

Now let's see what Bob and Betty are doing! Look at the next page.

MEASUREMENT AND DIMENSIONS

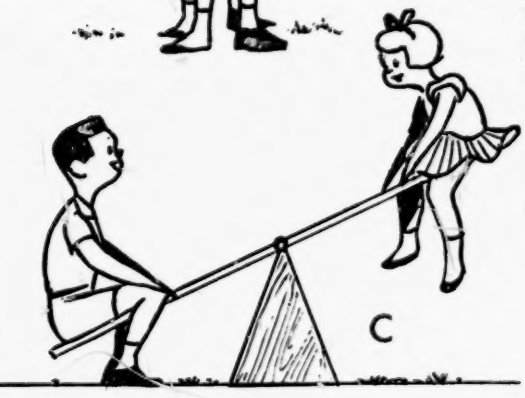
Who Is Taller? Who Is Heavier?

A

B

1. What do you suppose Betty and Bob are doing? Let's listen to them. "I'm older than you are," says Betty. "Yes," answers Bob, "you're older, but I'm taller. And I'm heavier, too. I'm large for my age. You're small for yours." How can Betty and Bob settle their discussion? Look at Pictures B and C to see what they are doing.
2. How are Betty and Bob comparing their heights?
3. How are Betty and Bob comparing their weights?
4. Who is taller? Heavier? How do you know?
5. In what other ways can Betty and Bob compare their sizes?

Turn to the next page to check your thinking. You will also find there some suggestions for things you may want to do.



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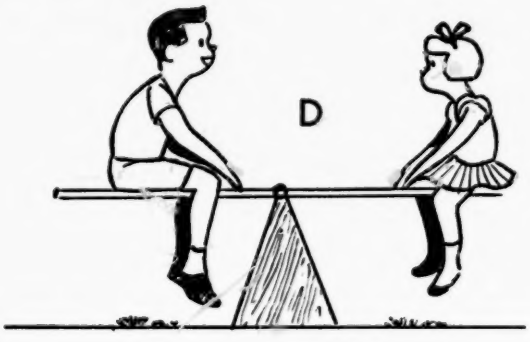

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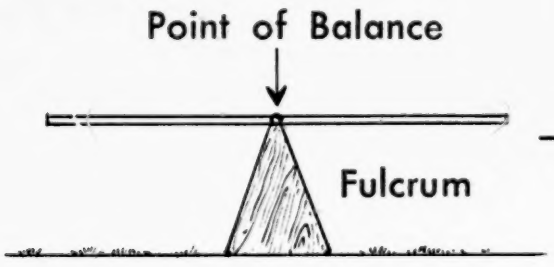


The point of balance is at the center of this seesaw. When the seesaw is at rest it remains in a level position.

CHECKING YOUR THINKING

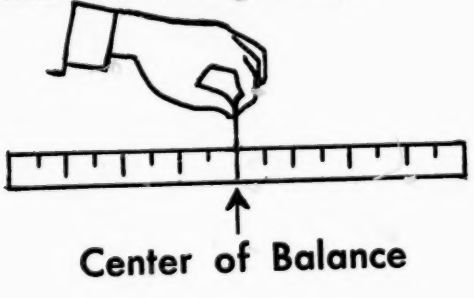
1. Betty and Bob are talking about which one of them is taller and which weighs more.
2. Betty and Bob stand back to back and compare directly to see who is taller.
3. Betty and Bob get on a seesaw (a teeter-totter) to see who is heavier. They are comparing their weights by balancing on the seesaw.
4. It is easy to see that Bob is taller, isn't it? Can you tell for sure that Bob is heavier? Yes, if you have ever played on a seesaw you know that children of equal weights balance when they are equally distant from the center. When one child is heavier than the other, his end of the seesaw goes down. Which end of the seesaw is down in Picture C on page 3?

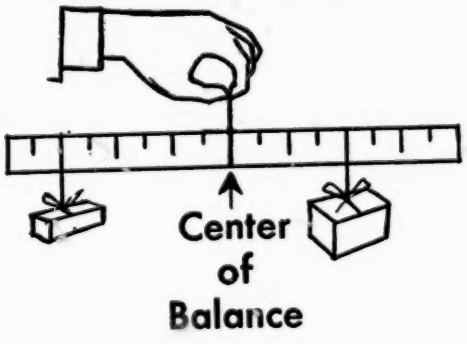
   A child of lighter weight can balance a child of heavier weight by sitting farther from the center. In Picture D who is farther from the center of the seesaw, Bob or Betty? Betty is balancing Bob's heavier weight by sitting farther from the point of balance.

   Bob and Betty can compare the widths of their shoulders, the lengths of their arms, the distances around their waists, the sizes of their hands and their feet. Can you think of other comparisons they might make?

FINDING OUT ON YOUR OWN

Do you like to experiment? You may want to try balancing a yardstick. You can locate the center of balance by balancing the yardstick across your finger. Now put a string around it at the center of balance. See if you can make something light balance something heavy.





Do you like to find information in books? You may want to find out more about balancing things by learning about levers and the "law of the lever." You can find out about levers and how they work in these books and others:

Machines, by Bertha Morris Parker

Simple Machines and How They Work, by Elizabeth N. Sharp

What Is a Simple Machine? by Gene Darby

Science Far and Near, by Herman and Nina Schneider

Do you like to find out about words? What does the dictionary tell you about these words?

compare	measure	balance	center
fulcrum	pivot	estimate	level

You might want to make a booklet of words that have to do with comparing and measuring. You can simply make a list of the words, or you can put them into sets of words that go with certain ideas. An example of ways that you can put words into groups is shown below. Some words might belong in more than one list. We could say that the sets of words overlap. For example, we say both a **long** distance and a **long** time, don't we? Watch for words for your booklet in many things which you read.

Size	**Length and Distance**	**Speed**	**Time**
big	long	fast	now
little	short	slow	then
tiny	far	rapid	long ago
Weight	**Temperature**	**Position**	**Shape**
heavy	warm	where	circle
light	cool	center	square
pound	icy	level	polygon

Comparing and Measuring

Every year Bob exchanges letters with his cousin John, who lives in England. The boys tell each other all sorts of things about themselves.

Bob thinks, "I'd like to know who is larger, John or I. We're about the same age. I'll tell him that I'm larger than Betty, and ask him about my size compared to his."

Suppose Bob writes, "I'm larger than Betty. How does my size compare to yours?" Could John answer Bob's question? Why not? That's right, he needs more information. While he knows how Bob's size compares to Betty's, he doesn't know how Betty's size compares to his.

When Bob thought more about his letter he said, "That's a silly question to ask John. He couldn't answer it, and besides I know the answer. Mother said that in most ways Betty is larger than John. If I am larger than Betty and she is larger than John then I am larger than John."

"What I really want is more definite information about my size compared to John's. I expect that I'm heavier and taller than John. How much heavier and taller? Are my shoulders wider than his? Are my arms longer? I'll have to measure myself and send him the information."

Bob will find his weight in number of pounds and his height, width of shoulders and length of arms in numbers of inches. Then John can compare these measures to his own weight, height and so on. He can do so because he knows what a pound is and what an inch is, and understands how to work with numbers.

The **pound** and the **inch** are **units of measure** which almost everyone has agreed upon. People know what these units mean. Most of the units of measure used in the United States are the same as those used in England. John, who lives in England, has learned them just as you and Bob have learned them.

The **distance** from point A to Point B is 1 inch.

Can You Do It?

Bob had reasoned this way:

If I am larger than Betty and Betty is larger than John, then I am larger than John.

Try to answer the following questions by reasoning. Write your answers on a sheet of paper. If you have trouble, drawing a picture may help you. Check your answers by reading **Checking Your Thinking** on this page.

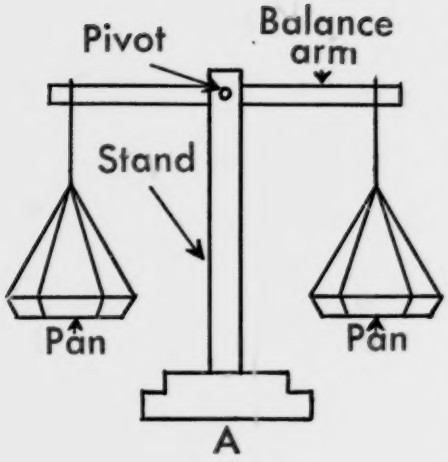
1. Which of the three boys can swim the fastest? You know that Gary can swim faster than Jim and Jim can swim faster than Andy.
2. Which of the three girls is the tallest? You know that Ann is taller than Susan and Linda is taller than Susan.
3. What is the least number of girls who can dance in a line arranged so that there are 2 girls in front of a girl, 2 girls behind a girl, and a girl between 2 girls?
4. If Sam has saved less money than Jack and Bill has saved more money than Sam, which boy has saved the least?
5. If Sarah lives twice as far from school as Tom and he lives half as far as Carol, which child lives nearest to school?
6. If Alice has practiced for 10 minutes and this is $\frac{1}{3}$ of the time she will practice, how much time will she practice all together?
7. If Pat's mother is 4 times as old as Pat and is 32 years old, how old is Pat?

CHECKING YOUR THINKING

1. Gary can swim fastest.
2. There is not enough information given to answer the question. You need to be able to compare Ann and Susan.
3. Three girls.

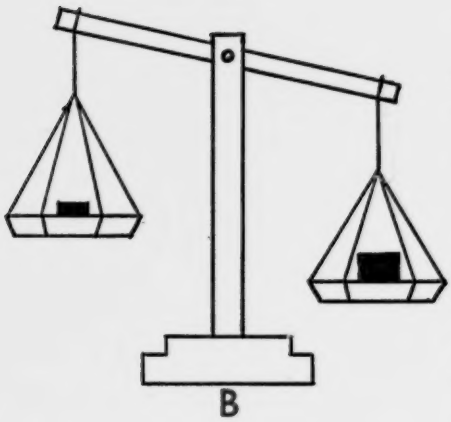
4. Sam has saved the least money.
5. Tom lives nearest to school. Did you figure out that Carol and Sarah live the same distance from school?
6. If $\frac{1}{3}$ of the time is 10 minutes, then Alice will practice 3 x 10 or 30 minutes.
7. Pat is 8 years old.

Balance and Scales

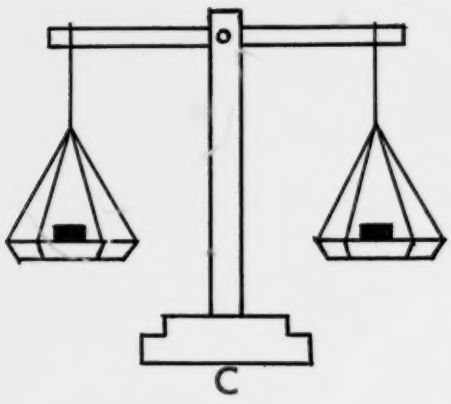


Have you ever seen a balance similar to the one in Picture A? It has an upright stand and a balance arm which is pivoted at its center. Pans are attached near the ends of the balance arm.

A balance can be used to compare the weights of two objects by placing one object in one pan, the other object in the other pan. How can you tell when one object is heavier than another? That's right, the pan with the heavier object will move downward while that with the lighter object moves upward (Picture B). If the objects are equal in weight the arm of the balance will become level when it is at rest (Picture C).

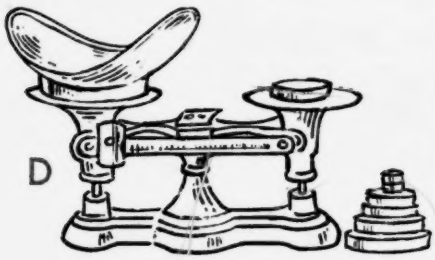


Do you remember how unequal weights can be made to balance each other on a seesaw? The pans on this balance cannot be moved along the arm, so the weights must be equal in order to balance. The pull of the earth on the objects is equal when the arm of the balance is level. We use the pull of the earth to measure weight when we compare weights by balancing.



A balance is usually called **scales** when the weight of one object or set of objects is known and is used as a basis for comparing the weight of another object.

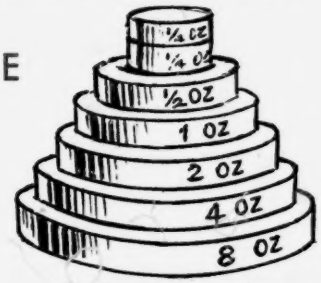
Picture D shows one kind of balance scales and several weights. The object to be weighed is placed in the pan. Enough weights are placed on the round platform at the opposite end of the arm to put the scales in balance. Each weight is labeled.



D

In Picture E the unit of weight is the ounce, and the number of ounces of each weight is shown. The measure of the weight of the object is the total number of units of weight needed to balance it.

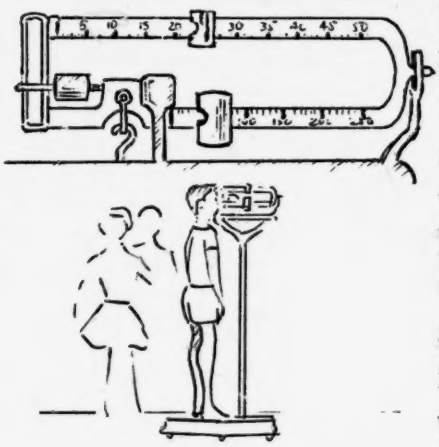
Scales of this type are often used to weigh such small objects as candy, nails, or bolts. Pharmacists and bakers also use them in their work. There might be 7 weights in a set.



E

What is the largest amount that could be weighed with a set of weights like that in Picture E? That's right, 1 pound. There are 16 ounces to a pound. The abbreviation for ounce is **oz.**, and **lb.** is the abbreviation for pound.

Balance scales of the type on which Bob is weighing himself use the same principle as the balance in Pictures A through D. When Bob stands on the platform his weight is very close to the center of balance, so a small weight on the weigh arm can be used to balance his larger weight. Although the scale doesn't look like a seesaw, it uses the same principle that made it possible for Betty to balance Bob's larger weight by moving farther from the center of balance of the seesaw.



The weights on the weigh arm are moved to the point that keeps the weigh arm free. The arm does not rest on the bar below it nor touch the bar above. The amount of the weight is read at the point on the weigh arm where the weight is placed. The numerals on the weigh arm show pounds and fractions (parts) of pounds. Can you read Bob's weight? The measure of his weight in pounds is $72\frac{1}{2}$.

FINDING OUT ON YOUR OWN

Do you like to keep scrapbooks or notebooks? You may want to make a book about measurement and the tools (instruments) for measuring. Part of it could be about weight and scales. Part could be about length and rulers, yardsticks or tapes. Part could be about time and sundials, clocks, and watches. You may think of other parts for your book. You might find pictures in books, magazines and newspapers. You might draw pictures of things you have seen, copy pictures from books and cut out pictures from magazines and newspapers for your book. You might label the pictures and write explanations about them.

Do you like to experiment? You may want to see how closely you can estimate weights. To estimate means to think about a length, a weight or another measure and to make your best judgment about it.

Ask your mother to let you lift a pound can full of coffee, a pound of butter or margarine, and other things that weigh a pound. Look at the labels to be sure that the weight of each is a pound, and check to be sure that the package is full. Remember that **lb.** is the abbreviation for pound and **oz.** is the abbreviation for ounce. Practice lifting these until you think that you have the "feel" of a pound.

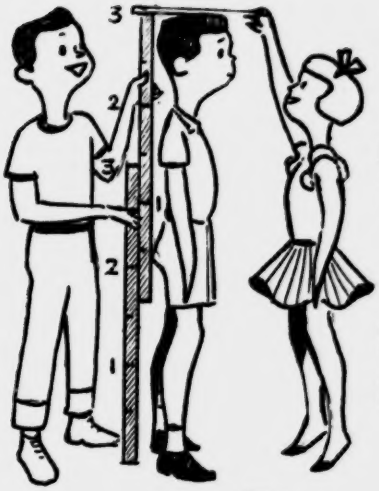
Lift other packages and estimate their weights. Try packages of dry cereal and packages of flour or cake mix. Which is larger? heavier? Find the weights on the labels.

Do you like to feel sure of your knowledge? A table is a set of information arranged in an orderly way so that relationships can be seen easily. The table below gives the names of some units of weight, their abbreviations, and the relation of one unit to another.

Table of Units of Weight

16 ounces (oz.)	=	1 pound (lb.)
100 pounds	=	1 hundredweight (cwt.)
2000 pounds	=	1 ton (T.)

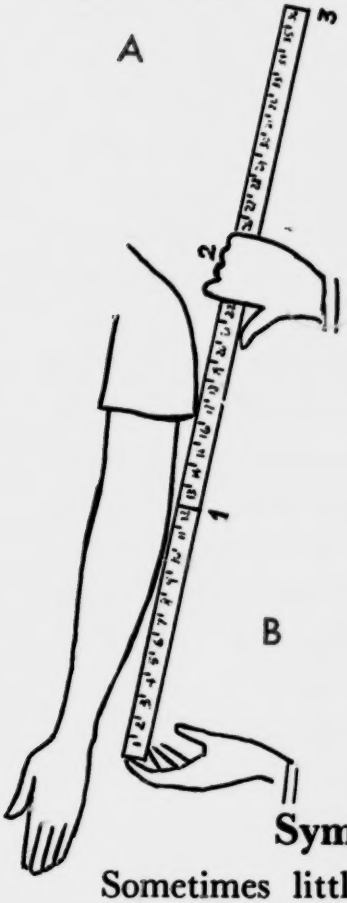
You do not add an s to the abbreviations; **oz.** stands for both ounce and ounces; **lb.** stands for both pound and pounds.



How Tall?

Betty and a friend are helping Bob measure himself.

1. What instrument of measure are the children using in Pictures A and B? What else could they use?
2. Estimate how tall you think Bob is.
3. Estimate the length of Bob's arms.



Linear Units

Have you ever thought that when we measure length, width, height, depth, and so on we are really measuring distance from one point to another point on an imaginary line? Such measures are called linear measures. Copy the following sentences and see if you can complete them. You can use a yardstick and a ruler to find the answers for Exercises 1 through 6, and the table of linear units on the following page for Exercises 7 and 8. Check your thinking on page 12.

1. 12 inches =
2. 36 inches =
3. 3 feet =
4. $\frac{1}{2}$ yard =
5. $\frac{1}{4}$ yard =
6. $\frac{3}{4}$ yard =
7. $\frac{1}{4}$ mile =
8. $\frac{1}{2}$ mile =

Symbols for Inches and Feet

Sometimes little marks or signs are used instead of the words or abbreviations for feet and inches. These marks are called **symbols.**

This symbol (′) means foot or feet. 3′ is read **3 feet.**

This symbol (″) means inch or inches. 4″ is read **4 inches.**

1. Copy the numerals. Write the correct abbreviation in place of each symbol for inches or feet. Use the table below if you do not know the abbreviations. Check your thinking at the bottom of this page.

a. 3′	**b.** 5″	**c.** 6′	**d.** 8″
e. 2′ 4″	**f.** 4′ 6″	**g.** 9′ 3″	**h.** 7′ 10″

2. Copy the numerals. Write the correct symbol in place of each abbreviation.

a. 11 in.	**b.** 2 ft.	**c.** 18 ft.	**d.** 20 in.
e. 1 ft. 2 in.	**f.** 4 ft. 6 in.	**g.** 6 ft. 4 in.	**h.** 10 ft. 9 in.

Table of Linear Units

12 inches (in.)	=	1 foot (ft.)
3 feet (ft.)	=	1 yard (yd.)
5280 feet (ft.)	=	1 mile (mi.)

CHECKING YOUR THINKING

1. Bob's friend is using two yardsticks to measure Bob's height. Betty is holding a ruler to help. The children might also have used a measuring tape.
2. Did you look at the yardsticks and think somewhat as follows? "One yardstick is 3 feet long. Bob's height is more than another foot of the second yardstick. He is close to $4\frac{1}{2}$ feet tall."
3. 22 inches (close to 2 feet).

Linear Units

1. 1 foot	2. 1 yard	3. 1 yard	4. 18 inches
5. 9 inches	6. 27 inches	7. 1320 feet	8. 2640 feet

1. a. 3 ft.	**b.** 5 in.	**c.** 6 ft.	**d.** 8 in.
e. 2 ft. 4 in.	**f.** 4 ft. 6 in.	**g.** 9 ft. 3 in.	**h.** 7 ft. 10 in.
2. a. 11″	**b.** 2′	**c.** 18′	**d.** 20″
e. 1′ 2″	**f.** 4′ 6″	**g.** 6′ 4″	**h.** 10′ 9″

Using a Weight - Height - Age Table

Bob is 8 years old, 53 inches tall, and weighs $72\frac{1}{2}$ pounds. He says that he is tall and heavy for his age. You can use a Weight-Height-Age Table for boys to see if Bob is right.

The word **data** means a collection of facts. The data for the tables on pages 14 and 15 were obtained by weighing and measuring many, many boys and girls.

First, let's find out how to use the tables. Look at the table for boys.

1. Do you see the row across the top that is labeled **Age in Years**? Bob's age is 8 years, so find the column headed 8.
2. Look down the column at the left of the double line to find Bob's height, which is 53 inches.
3. Run your finger across from 53 to the numerals in the column headed 8. Do you see 62 - 74? These numerals tell us that most boys who are 8 years old and 53 inches tall weigh somewhere from 62 to 74 pounds. The **range** of their weight is from 62 to 74 pounds.

Bob's weight is $72\frac{1}{2}$ pounds. We say that his weight is within the **range** for his age and height. $72\frac{1}{2}$ is between 62 and 74.

What other weights would fall within this range? Yes, 64, $64\frac{1}{2}$, 65, $65\frac{1}{2}$ and so on to 74 pounds. Of course, some boys 8 years old and 53 inches tall weigh more and some weigh less than the weights in this range. Most, however, will be somewhere within the range.

WEIGHT - HEIGHT - AGE TABLE FOR GIRLS

Height in Inches	Age in Years 7	8	9	10
41	36-43			
42	37-44			
43	38-46	38-46		
44	40-48	40-48		
45	42-50	43-50	43-52	
46	45-53	45-54	45-54	45-56
47	46-55	46-56	46-57	46-58
48	48-58	48-58	48-59	48-60
49	51-61	51-62	51-62	51-64
50	53-63	53-64	53-66	54-67
51	55-67	55-67	56-69	56-70
52	58-71	58-72	58-72	58-73
53	61-73	61-74	62-76	62-78
54		63-77	64-79	64-81
55		66-81	67-82	67-84
56			70-86	70-88
57			73-89	73-91
58				76-95
59				80-100
60				84-105

WEIGHT - HEIGHT - AGE TABLE FOR BOYS

	Age in Years			
Height in Inches	7	8	9	10
41	37-44			
42	38-45	39-46		
43	39-46	40-47		
44	41-48	41-49	42-51	
45	43-50	43-52	44-53	
46	45-52	45-54	46-55	
47	47-55	47-56	48-57	48-58
48	49-58	49-58	50-59	50-60
49	51-60	52-62	52-62	52-63
50	54-63	54-65	54-65	55-66
51	57-66	57-68	57-68	57-69
52	60-70	60-71	60-71	60-72
53	62-73	62-74	63-75	63-75
54		63-76	64-77	64-78
55		65-78	67-80	67-81
56			70-84	71-85
57			73-87	74-89
58			76-91	77-93
59				80-93
60				84-101

Is Bob tall and heavy for his age? Let's see how you can use the table to find out. Locate the numerals 62 - 74 again at the point where the column for Bob's age and the row for his height meet.

How many pairs of numerals are there in this column above 62 - 74? That's right, 11. How many are there below this point? Yes, 2. The list of weights for age 8 ends at the row for a height of 55 inches. We could say, then, that Bob is among the taller and heavier boys of his age, although there are boys who are taller and heavier.

1. Can you find the range of weights given for boys 9 years old who are 51" tall? First find the age at the top of the table for boys. Next look down the side column to find the height, in this case 51". Place your finger on the place where the row and column meet, and read the range of weights listed. Check your thinking on page 17.

2. Find the range of weights given for girls who are 9 years old who are 51" tall. Use the table for girls on page 14. Check on page 17.

3. If you know your age, height and weight, you can compare your weight to the range which is given for your age and height. Boys should use the table on page 15 and girls should use the table on page 14.

Don't worry if your weight does not come within the given range for your height and age. While the weights of a majority of children of your age and height fall within this range, the weights of many are above or below the range.

4. Can you figure out how to find the range of heights shown for boys who are 7 years old? 8 years old? 9 years old? 10 years old? Check your thinking on page 17.

CHECKING YOUR THINKING

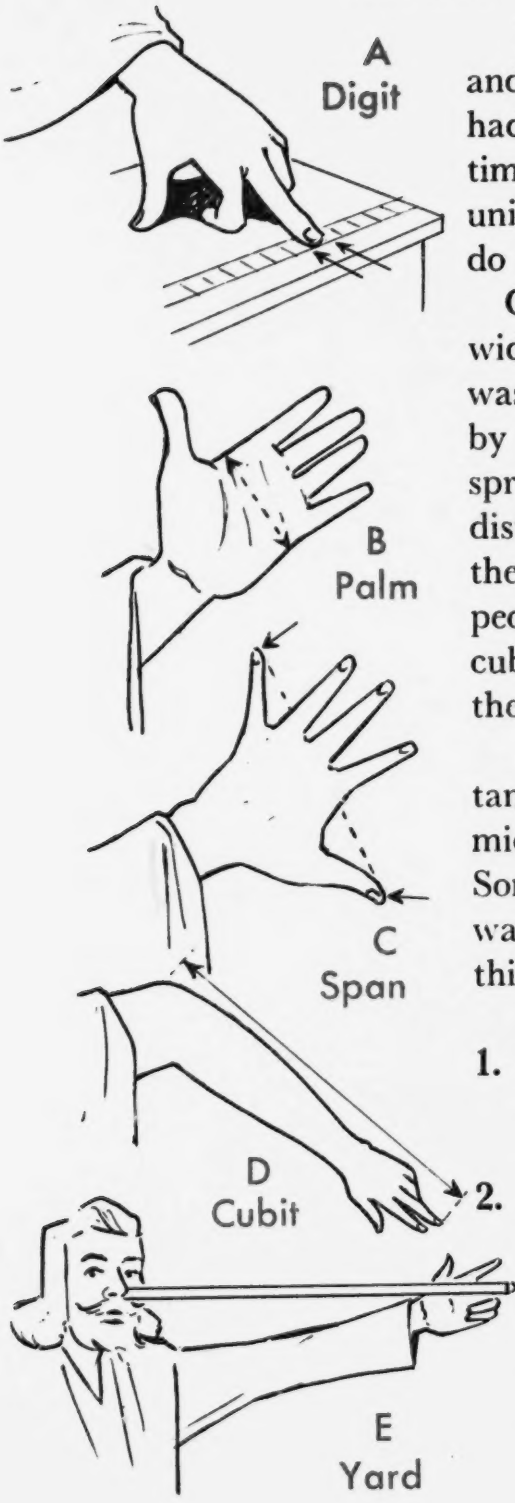
1. The range of weights of boys 9 years old who are 51″ tall is from 57 to 68 pounds by the table on page 15.
2. The range of weights of girls 9 years old who are 51″ tall is from 56 to 69 pounds by the table on page 14.
3. The answer depends of course on your age, height, and weight.
4. Find the column headed **Height in Inches,** and then find the columns labeled **Age in Years.** There is a range of weights given for a height of 41 inches, and one for each height to 53 inches. The range of heights for 7 years is 41-53 inches. For 8 years the range is 42-55, for 9 years it is 44-58 and for 10 years it is 47-60 inches.

FINDING OUT ON YOUR OWN

Have you wondered about variations in heights and weights? You might want to read parts of the books which are listed below. Look in the Table of Contents or the Index of these books for such topics as growth, height, weight, and so on.

Author	Title
Bauer, W. W. and Others	**About Yourself**
	From Eight to Nine
	Going On Ten
Beim, Jerrold	**The Smallest Boy in the Class**
Brownell, Clifford L. and Others	**About Your Health**
	Building Better Health
Burkard, William E. and Others	**All Aboard for Health**
	Better Health for You
Charters, W. W. and Others	**Healthful Ways**
	Let's Be Healthy
Hallock, Grace T. and Allen, R. L.	**Growing Your Way**
	Keeping Healthy and Strong
Jones, Edwina and Others	**Health Trails**
	Your Health and You
O'Keefe, Pattric R. and Others	**How We Grow**

Measuring Long Ago



Long ago people did not have rulers and yardsticks or scales. In fact, they had no measuring instruments. Sometimes they used parts of their bodies for units for measuring things. Do you ever do that?

One way people measured was by the width of the **finger** or digit, another way was by the **palm,** and another way was by the **span.** To measure by a span, spread your hand wide. The span is the distance from the tip of the thumb to the tip of the little finger. A fourth way people measured was by the **cubit.** The cubit is the distance from the elbow to the tip of the longest finger.

An old measure of **yard** was the distance from the nose to the tip of the middle finger of the outstretched arm. Some people still estimate yards in this way. Have you ever seen anyone use this way of measuring?

Here are some things for you to do.

1. Measure your desk by spans. How many spans long is it? How many spans wide?
2. Compare the length of your span to the spans of several friends and to your mother's and father's spans. Would all of you get the same number of spans if each measured your desk by spans? After you think about your answer, check your thinking on page 19.

3. Suppose you want to buy ten spans of ribbon, would you rather buy it from a grown person like your mother or father or from a child? Why? Check your thinking at the bottom of the page.
4. Can you make a table of related units of measure from the finger, palm, span, cubit and yard? Get a paper ready to record your work. Measure your palm by your finger width, your span by your palm, your cubit by your span, and your yard by your cubit. After you have made a record of your results, check your thinking below.
5. Measure your span, cubit and yard with a ruler or yardstick. How many inches is each? Is your span more or less than a foot? Your cubit? Your yard?

   Perhaps you can measure your father's and mother's cubit and yards with a ruler or yardstick. See if their spans are close to 9 inches, their cubits close to 18 inches, and their yards close to 36 inches in length.

CHECKING YOUR THINKING

1. The answer depends on the size of your span and the size of your desk.
2. Probably your friends' spans are closer to yours in size than your mother's and father's are. Of course, the smaller the span the larger the number of them it will take to measure your desk. You and your father would certainly get a different number of spans if both measured your desk by spans. His span is larger so he would get a smaller number of spans for his measure of your desk.
3. You would get more ribbon if you bought 10 spans from the person with the largest span. You'd probably get more by buying it from a grown person, wouldn't you?
4. The table for most people is

   4 fingers = 1 palm
   3 palms = 1 span
   2 spans = 1 cubit
   2 cubits = 1 yard
5. The answer will depend upon the sizes of the people you measure.

FINDING OUT ON YOUR OWN

Do you like to find interesting information? You can find information about measures in encyclopedias. Look for such topics as the following: Measures, Measurement, Metric System, National Bureau of Standards, Precision, Weights and Measures.

You can write for information to the companies in the following list. Tell them that you are learning about measurement and ask them to send you any information which would be helpful to you. Ask your teacher to put her signature on your letter below your own.

Ford Motor Company, Education Division,
The American Road, Dearborn, Michigan
General Motors, Department of Public Relations,
Detroit 2, Michigan
The Lufkin Rule Company, Saginaw, Michigan

You can learn about measuring by reading books of various types. You may want to ask for these books at your school library or your public library.

Author	Title
Bell, Thelma Do snowflakes have any relationship with geometry?	**Snow**
Blough, Glenn O. How do things grow? What do measures have to do with growing things?	**Wait for the Sunshine**
Earle, Oliver L. How long have camels and llamas lived on earth?	**Camels and Llamas**
Freeman, Mae and Ira Do you want to learn about the stars?	**Fun with Astronomy**
Green, Erma How is steel made? What is the temperature of the furnace? How many tons can it hold?	**Let's Go to a Steel Mill**

Author	Title
Larson, Harold D. Why did people need measures? What things did they use for measuring?	**The Story of Measure**
Lerner, Margaret R. What is a sphygmomanometer?	**Doctor's Tools**
Malter, Morton S. Which animals are largest? Smallest? How do people find out about them?	**Our Largest Animals, Our Tiniest Animals**
McCullough, John G. and Kissler, Leonard Would you like to know how people have developed ways to travel farther and faster?	**Farther and Faster**
Moore, Lillian Do you know what a plumb line is?	**The Important Pockets of Paul**
Parker, Bertha Morris There are special sections on time, size, rate, distance, and temperature.	**The Golden Book of Science**
Shannon, Terry Why are measurements important in making ready-to-wear clothes?	**About Ready-to-Wear Clothes**
Smith, George O. Why is mathematics so important to the scientist?	**Mathematics, the Language of Science**
Sootin, Laura How are measures used at an airport?	**Let's Go to the Airport**
Watson, Nancy D. Do you like to spend money? Do you know the values of coins?	**Annie's Spending Spree**

Measuring In Modern Times

Suppose that Bob didn't measure his height in inches but instead wrote to John, "My height is 11 spans." Would this information help John to know Bob's height?

It might help a little if John knew the span of Bob's hand compared to his. However, does Bob's statement, "My height is 53 inches," give John a better idea? Yes, because John knows the length of an inch and can use a ruler, yardstick, or tape to make measurements.

People can depend upon units of measure today. The inch, the pound, and other units by which we measure things are established by law in most countries. The Constitution of the United States gives Congress the power "to fix the standards of weights and measures."

The system of measures we use is the English system, because the United States is an English-speaking nation. The English system of measures developed very slowly through hundreds of years of time. The units came from the body measures we just described and from things which were in the natural surroundings of the people. The relationships between these units were not planned ahead of time.

The metric system, however, was carefully planned. It is easy to change from one metric unit to a larger or smaller one because the units are related by tens, like our decimal system for writing number values.

You know that in our way of writing numerals we think of the ones' place as the **basic place value** and relate the other place values by multiplying or dividing by ten. Ones' place is also called units' place. In the table below **x** means **multiplied by** and $\div$ means **divided by.**

Decimal Numerals

100	$= 1 \times 100$ or 10×10
10	$= 1 \times 10$
1	(Basic Unit)
0.0	$= 1 \div 10$ or $\frac{1}{10}$
0.01	$= 1 \div 100$, or $\frac{1}{100}$, or $\frac{1}{10} \div 10$

In the metric units for linear measure the meter is the basic unit. Other linear units are related to the meter by multiplying or dividing by ten.

Some Metric Units of Length

Hectometer	100	= 1 meter x 100 or 1 decameter x 10
Decameter	10	= 1 meter x 10
Meter	1	(Basic Unit)
Decimeter	0.1	= 1 meter ÷ 10 or $\frac{1}{10}$ of a meter
Centimeter	0.01	= 1 meter ÷ 100, or 1 decimeter ÷ 10, or $\frac{1}{100}$ of a meter

Metric units for weight and for other kinds of measuring are related by tens in the same way as the linear units. Units for different kinds of measuring also are related to each other in an orderly way.

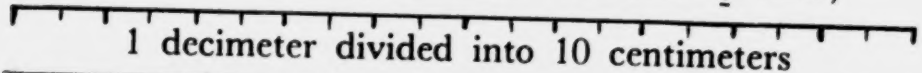
In the United States our units are defined in relation to metric units. Scientists in our country usually use the metric system and some manufacturers use it.

You may want to know how the units of the metric system which you are likely to see in use compare with units that you know.

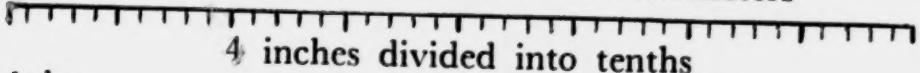
Some Metric Units Compared to English Units

Linear Units

Meter	About 3 inches longer than a yard
Centimeter	About $\frac{4}{10}$ inch (almost $\frac{1}{2}$ inch)
Kilometer	About $\frac{6}{10}$ mile (a little over $\frac{1}{2}$ mile)



1 decimeter divided into 10 centimeters



4 inches divided into tenths

Weight

Gram	About 0.034 ounce (a dime weighs about 2 grams)
Kilogram	About $2\frac{2}{10}$ pounds

Capacity

Liter	A very little more than 1 liquid quart

The science room in your school may have a meter stick, scales with weights in grams, and a ruler marked in centimeters. See if you can get these to examine.

Try the following problems for fun. Then read **Checking Your Thinking** at the bottom of the page.

1. Speedometers on some foreign cars measure speed in kilometers per hour. Suppose that you are riding in such a car. The speedometer shows that you are going at a speed of 100 kilometers per hour.
   a. Is 100 kilometers per hour as fast as 100 miles per hour?
   b. About what is your speed in miles per hour?
2. Some running races are in metric distances. All races of the Olympic games are in meters. Suppose that you are to run a 100-meter race. Will that be more or less than a 100-yard race?
3. Gasoline is sold by the liter in Mexico and in European countries. If the gas tank of a small car holds 10 gallons of gasoline, about how many liters will it hold?
4. If you wanted to have about 2 pounds of meat, how many kilograms of meat would you buy?

CHECKING YOUR THINKING

1. a. No.
   b. Since a kilometer is about $\frac{6}{10}$ of a mile, 100 kilometers per hour is about 60 miles per hour.

$$\frac{6}{10} \times \frac{100}{1} = \frac{600}{10} = 60$$

$$\begin{array}{r} 100 \\ \times .6 \\ \hline 60.0 \end{array}$$

2. More than 100 yards because a meter is longer than a yard. A meter is close to 39.37 inches. A yard is 36 inches.
3. Less than 40 liters. A liter is a little more than a quart. 1 gallon = 4 quarts. $4 \times 10 = 40$ quarts to a 10 gallon tank.
4. 1 kilogram, because 1 kilogram is a little more than 2 pounds.

A Different Way To Write Dates

When Bob wrote to John about their sizes, he wrote the date this way:

1 / 12 / 61

What do you think the numerals might stand for? Look at the date below and compare it to the one above.

January 12, 1961

In **1 / 12 / 61** the **1** at the left stands for January, because January is the first month of the year. The **12** stands for the twelfth day of the month, and the **61** stands for the year **19**61. You need to know the order of the months if you are to read and write dates in this way.

Try to write on a piece of paper the names of the twelve months in correct order. Check your thinking on page 26.

Be sure your list is correct. Now, number the months starting with **1** for January, **2** for February, **3** for March and so on. Check your thinking on page 26.

Try writing the following dates in the short way, like we wrote **1/12/61** to mean January 1, 1961. Copy the usual way and write the short way beside it.

1. February 22, 1960
2. October 3, 1962
3. June 19, 1961
4. December 25, 1962
5. January 1, 1963
6. July 4, 1965
7. March 21, 1961
8. August 16, 1963
9. May 10, 1962
10. November 11, 1964
11. April 10, 1965
12. September 28, 1963

Are any of the dates above holidays? Write the names of the holidays you find beside the dates. Check your thinking on page 26.

A Table of Units of Time

(Abbreviations shown in parentheses)

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (da.)
7 days	=	1 week (wk.)
30 days	=	1 month (mo.) for figuring
12 months	=	1 year (yr.)
365 days	=	1 year, except that 366 days = 1 leap year

CHECKING YOUR THINKING

Numbers for Months in Order

1 January	**4** April	**7** July	**10** October
2 February	**5** May	**8** August	**11** November
3 March	**6** June	**9** September	**12** December

Dates

1.	February 22, 1960	2/22/60	Washington's Birthday
2.	October 3, 1962	10/3/62	
3.	June 19, 1961	6/19/61	
4.	December 25, 1962	12/25/62	Christmas
5.	January 1, 1963	1/1/63	New Year's Day
6.	July 4, 1965	7/4/65	Independence Day
7.	March 21, 1961	3/21/61	
8.	August 16, 1963	8/16/63	
9.	May 10, 1962	5/10/62	
10.	November 11, 1964	11/11/64	Veteran's Day
11.	April 10, 1965	4/10/65	
12.	September 28, 1963	9/28/63	Yom Kippur

FINDING OUT ON YOUR OWN

Are you interested in the calendar? You might want to read some of these books:

Author	Title
Adler, Irving	**Time in Your Life**
Brindze, Ruth	**The Story of Our Calendar**
Eisenberg, A. L.	**The Story of the Jewish Calendar**
Galt, Tom	**Seven Days from Sunday**

How Old Are You?

Do people ever ask, "How old are you?" How do you answer? When you answer you probably tell the number of years of your age: "I am nine years old." "I am ten years old." But sometimes you need to give your age not only in years but in months too. Did you ever need to give your age in both years and months? Do you know how to figure your age in years and months?

This is the way Betty figured her age the last time she took an achievement test. An achievement test helps you compare your progress in school with that of thousands of children in many different schools. It is one way of measuring progress in school.

First Betty thought of the order of the months and gave a number to each month as you have just done.

Next Betty wrote the date the test was given and the date she was born. She used numbers for the months instead of their names so that she could subtract dates.

	Year	Month	Day
Date of Test	1960	10	12
Date of Birth	—1951	—3	—4
	9 years	7 months	8 days

When you figure ages in this way, you think of a month as having 30 days because this is an even number. If there are less than 15 days (half a month) in the difference of your subtraction you do not count them. If there are 15 days or more you count them as one more month. This is called rounding to the nearest month. One half a month is 15 days, so less than 15 days is not counted, while anything more than 15 days is counted as 1 whole month.

What would Betty do? Yes, she would say that her age was 9 years, 7 months at the time of the test.

Bob needed to write his age in years and months when he signed up for a swimming class. Here are the dates he used.

	Year	Month	Day
Date of Signing up	1961	5	20
Date of Birth	—1953	—3	—28

How can Bob subtract 28 days from 20 days? Yes, he can change one of the 5 months to 30 days and add it to the 20 days. Then his subtraction would look like that below.

		4	50
Date of Signing up	1961	5	20
Date of Birth	—1953	—3	—28
	8 yr.	1 mo.	22 days

Since 22 days is more than 15 days, Bob would round his answer and write his age as 8 years, 2 months.

Can You Do It?

Figure the ages at the test dates for the birthdates given below. Remember, you can change a month to 30 days and a year to 12 months if you need to do this for the subtraction. Check your thinking below.

	Date of Test	Date of Birth
1.	May 24, 1960	September 12, 1949
2.	February 28, 1959	February 18, 1950
3.	January 25, 1960	December 31, 1948
4.	May 17, 1961	June 2, 1952
5.	October 5, 1962	March 13, 1953

CHECKING YOUR THINKING

1. 10 years 8 months (and 12 days). Drop days because less than 15.
2. 9 years 0 months (and 10 days). Drop days because less than 15.
3. 11 years 1 month (and 4 days). Drop days because less than 15.
4. 8 years 11 months 15 days; because 15 days round to 1 month, age is 9 years (8 years plus 12 months).
5. 9 years 6 months 22 days; round to 9 years 7 months.

A Different Way To Tell Time

$2 \times 12 = 24$
military

Did you know that the armed services of our country have a different way of telling time than we ordinarily use? Betty and Bob's father showed them about military time.

A clock shows just 12 hours. However, we know that it takes two 12-hour periods to make one full day and night. Military time is based on counting straight through the 24 hours that start at midnight and end 24 hours later.

0001 stands for 1 minute after midnight.
On a submarine or aircraft carrier, when the time is 0001, servicemen call this "zero, zero, zero, one."

0010 stands for 10 minutes after midnight. What does 0045 stand for? That's right, 45 minutes after midnight.

0100 stands for 1:00 A.M. (A.M. means the morning, starting at midnight and continuing to noon.)

0925 stands for 9:25 A.M. This is called "zero nine twenty-five." What does 1130 stand for? Yes, 11:30 A.M.

1200 stands for noon, or 12:00 noon.

1300 stands for 1:00 P.M. because for military time you go on counting from 12:00 noon instead of starting over again the way we are used to doing. (P.M. stands for the hours from noon until midnight.) 1300 is called "thirteen hundred."

1430 is 2:30 P.M. or half past 2 o'clock in the afternoon. It is $1200 + 0230$. Add these and you get 1430 ("fourteen thirty").

What does 1815 stand for? That's right, it is the same time that we call 6:15 P.M.

Can You Do It?

See if you can change the following ordinary times to military times. Remember that A.M. means before noon and P.M. means afternoon. Check your thinking below.

1. 3:40 A.M.
2. 3:40 P.M.
3. 11:45 A.M.
4. 11:45 P.M.
5. 6:15 A.M.
6. 12:35 A.M.
7. 9:30 P.M.
8. 4:20 P.M.
9. 7:32 A.M.
10. Midnight

Now, see if you can change the following military times to our usual way of naming the hours of the day.

11. 2050 hours
12. 1025 hours
13. 2330 hours
14. 0008 hours
15. 2400 hours
16. 1400 hours
17. 2247 hours
18. 1318 hours
19. 0410 hours
20. 1750 hours

CHECKING YOUR THINKING

1. 3:40 A.M. = 0340 hours
2. 3:40 P.M. = 1540 hours
3. 11:45 A.M. = 1145 hours
4. 11:45 P.M. = 2345 hours
5. 6:15 A.M. = 0615 hours
6. 12:35 A.M. = 0035 hours
7. 9:30 P.M. = 2130 hours
8. 4:20 P.M. = 1620 hours
9. 7:32 A.M. = 0732 hours
10. Midnight = 2400 hours or 0000 hours

11. 2050 hours = 8:50 P.M.
12. 1025 hours = 10:25 A.M.
13. 2330 hours = 11:30 P.M.
14. 0008 hours = 12:08 A.M.
15. 2400 hours = midnight
16. 1400 hours = 2:00 P.M.
17. 2247 hours = 10:47 P.M.
18. 1318 hours = 1:18 P.M.
19. 0410 hours = 4:10 A.M.
20. 1750 hours = 5:50 P.M.

Are you interested in clocks and time? You might want to read the following books.

Author	Book
Behm, Harry	**All Kinds of Time**
Larsen, Harold D.	**The Story of Time**
Zarchy, Harry	**Wheel of Time**
Ziner, Feenie and Thompson, Elizabeth	**The True Book of Time**

Has The Calendar Ever Changed?

Did you know that in the year A.D. 1582 the day after October 4 suddenly became October 15?

This is the way it happened. In the year 46 B.C., many nations adopted the Julian calendar, named after Julius Caesar. You know that **B.C.** means before Christ; **A.D.** means Anno Domini, in the year of our Lord. It turned out that by the year A.D. 1582 this calendar was ten days fast. Pope Gregory XIII corrected the error by dropping 10 days from the Julian calendar and starting the use of a new calendar, named after him.

But this Gregorian calendar was not adopted by England and the American colonies until early in September, 1752. By that time (from 1582 to 1752) another day of error had been added, minute by minute over the years, until the old style calendar still in use was eleven days fast. September 3, 1752, was therefore called September 14, to correct the 11-day error. All dates before September 14, 1752, are now written with O.S. for **Old Style** (the Julian) calendar.

To change from the Old Style or Julian calendar to the present day Gregorian calendar add days as shown below.

10 days for the years 1582 to 1700
11 days for the years 1700 to 1800
12 days from 1800 to 1900
13 days since 1900

Subtract the same number of days to change from the Gregorian to the Julian (O.S.) calendar.

George Washington was born February 11, 1732 O.S. After 1752 his birthday became February 22. Can you find the O.S. birthdates for these other famous Americans?

1. Benjamin Franklin	January 17, 1706
2. John Adams	October 19, 1735
3. John Hancock	January 12, 1737
4. Thomas Jefferson	April 13, 1743

Check your thinking on page 32.

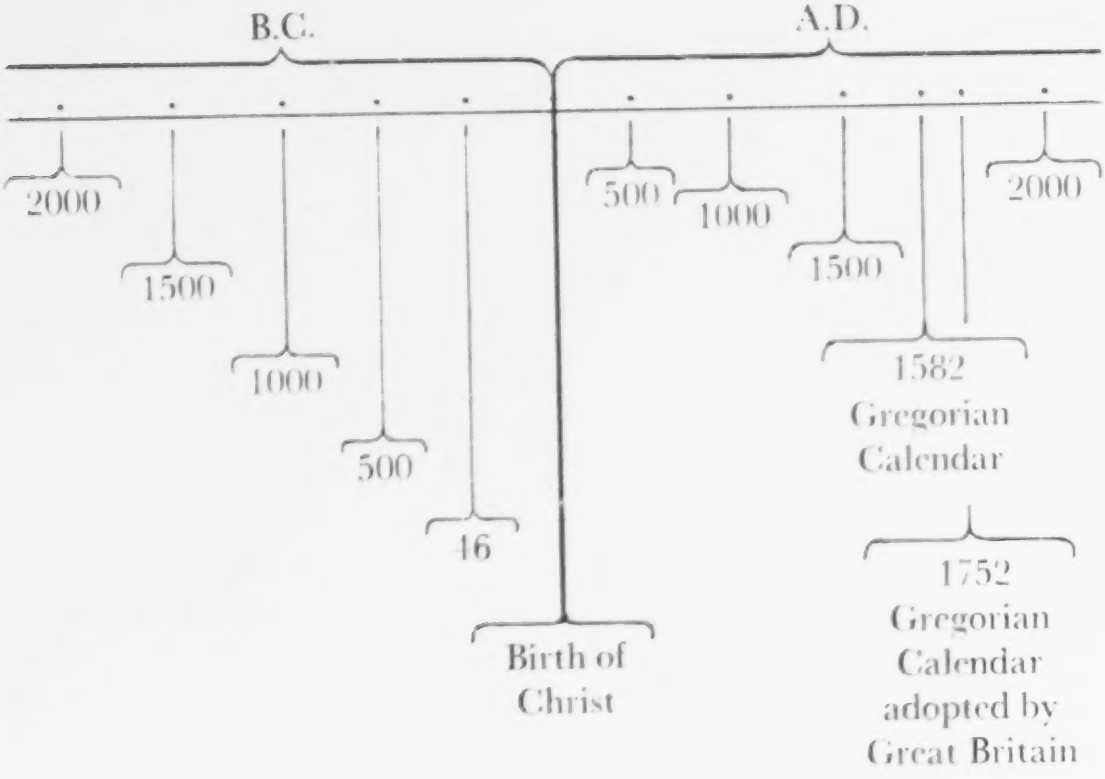
CHECKING YOUR THINKING

c a l e n d a r

	Gregorian	Old Style
1. Benjamin Franklin	January 17, 1706	January 6, 1706
2. John Adams	October 19, 1735	October 8, 1735
3. John Hancock	January 12, 1737	January 1, 1737
4. Thomas Jefferson	April 13, 1743	April 2, 1743

To find the number of years ago that the Julian calendar came into use, add 46 to the present year. For example, in 1961 the Julian calendar had come into use 1961 + 46 = 2007 years ago.

Our calendar begins with the birth of Christ. The time line may help you understand why you add to find the difference between 46 B.C. and the present. You think of the length of time from 46 B.C. to the present year.



To find how long ago the Gregorian calendar came into use you subtract 1582 from the present year. In 1961 the Gregorian calendar had been in use 379 years: 1961 − 1582 = 379.

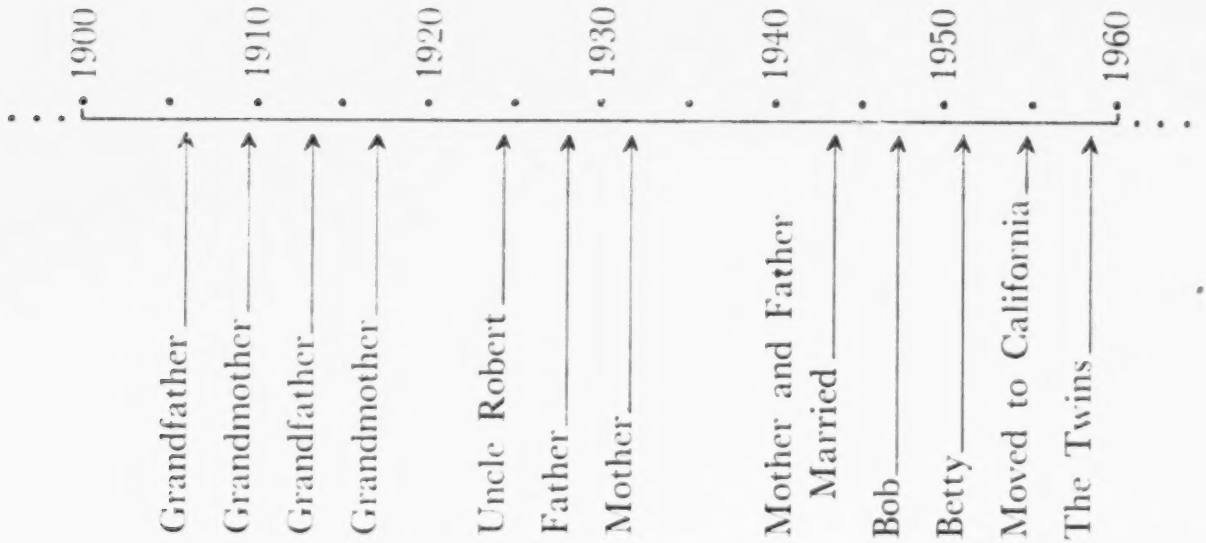
The Gregorian calendar came into use in America 209 years ago: 1961 − 1752 = 209.

FINDING OUT ON YOUR OWN

Are you interested in calendars? A plan has been made for a new calendar called the World Calendar. See if you can find some information about it in an encyclopedia. Do you think the plan is a good one?

Did the Time Line interest you? You might want to make a time line of events in your life or the life of your family. One that Bob and Betty made of family birthdates is shown below. The dots at the ends of the line mean "and so on." They show that this line continues in both directions. The part, or segment, shown here is all we need at this time.

Each point marks five years. Only the tens (decades) are labeled.



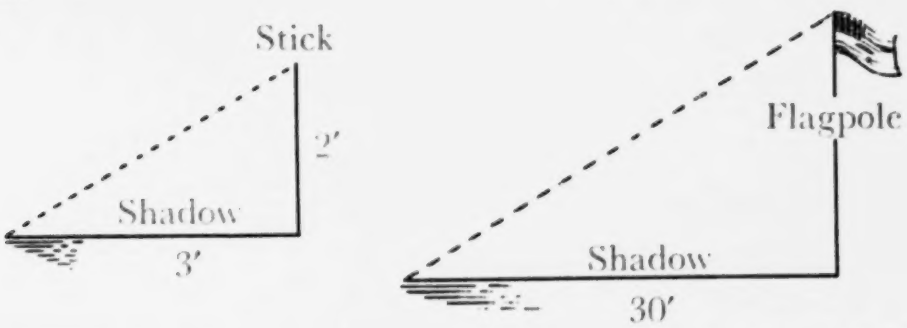
How Tall Is A Flagpole?

Have you ever wondered how tall the school flagpole is? A Boy Scout, a friend of Bob's, showed him an easy way to find out when you can't measure a pole directly.

Put a long straight stick into the ground near the flagpole. Watch carefully to see the shadows made by your stick and by the flagpole.

Since the light from the sun hits the stick and the flagpole from the same direction, both will have shadows falling in the same direction. Which shadow is longer?

Look carefully and you will see that you can draw an imaginary line connecting the top of the flagpole and the end of its shadow. Do the same for your stick. Do you see two triangles?



Measure the height of your stick and the lengths of the two shadows. Write down your figures. For example:

The height of the stick is 2′.

The length of the stick's shadow is 3′.

The length of the flagpole's shadow is 30′.

To find the height of the flagpole, do this:

1. Multiply the measure of the length of the flagpole's shadow by the measure of the height of the stick, $2 \times 30 = 60$.
2. Divide your answer by the measure of the length of the stick's shadow, $60 \div 3 = 20$.

If you have followed the directions carefully, your answer will be the height of the flagpole. The flagpole in our example is 20 feet in height.

Try the following examples for fun. They may help you to draw diagrams like those above. Check your thinking at the bottom of the page.

1. Find the height of a flagpole. Length of the flagpole's shadow is 42′, height of the stick is 3′, length of the stick's shadow is 6′.
2. Find the height of the tetherball pole. The tetherball pole's shadow is 8′, the height of the stick is 2′, the length of the stick's shadow is 4′.
3. Find the height of the tree. The tree's shadow is 45′, the stick's length is 2′, the stick's shadow is 6′.
4. What would the shadow of the pole have to be to give us a 10′ tetherball pole?

CHECKING YOUR THINKING

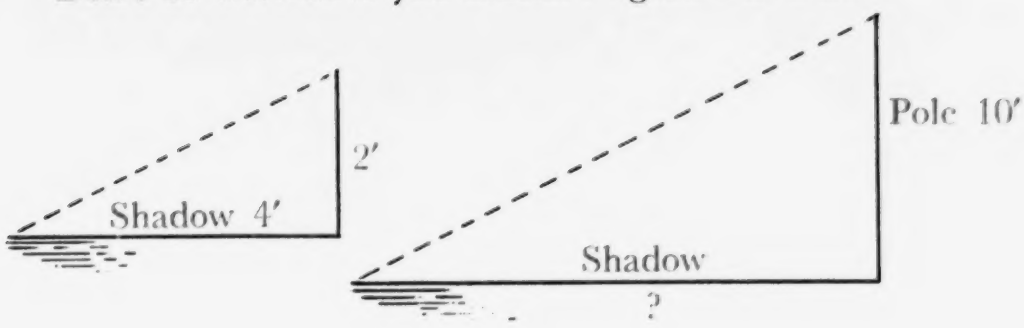
1. $3 \times 42 = 126$ $126 \div 6 = 21$ Height of flagpole is 21′
2. $2 \times 8 = 16$ $16 \div 4 = 4$ Height of tetherball pole, 4′
3. $2 \times 45 = 90$ $90 \div 6 = 15$ Height of tree, 15′
4. Does this one puzzle you? Thinking in this way may help you. What number multiplied by 2 will give a number that I can divide by 4 to get 10?

   The number that can be divided by 4 to get 10 is 40.
   20 multiplied by 2 gives 40.

   The pole's shadow must be 20 feet long.

   Try the number 20. Does it work?
   $2 \times 20 = 40$ $40 \div 4 = 10$

   Don't be worried if you couldn't figure this out!



Are you interested in discovering relationships?

1. Can you figure out some interesting relationships between the pairs of numbers in the examples about indirect measurement of height?

   a. Height of stick 2′ Height of pole 21′
   Stick's shadow 4′ Pole's shadow 42′
   What is the relation of 4 to 2? Of 42 to 21?

   b. Height of stick 2′ Height of pole 4′
   Stick's shadow 4′ Pole's shadow 8′
   How is 4 related to 2? 8 to 4?

   c. Height of stick 2′ Height of pole 15′
   Stick's shadow 6′ Pole's shadow 45′
   How is 6 related to 2? 45 to 15?

2. Now, look at just the number pairs.

   a. $\frac{2}{4}$ $\frac{21}{42}$ b. $\frac{2}{4}$ $\frac{4}{8}$ c. $\frac{2}{6}$ $\frac{15}{45}$

   Try "cross multiplying." Copy and multiply.

   a. $\frac{2}{4} \times \frac{21}{42}$ $2 \times 42 =$ $4 \times 21 =$

   b. $\frac{2}{4} \times \frac{4}{8}$ $2 \times 8 =$ $4 \times 4 =$

   c. $\frac{2}{6} \times \frac{15}{45}$ $2 \times 45 =$ $6 \times 15 =$

   Do you see an interesting relationship? For **a** were both products 84? For **b** were both products 16? For **c** were both products 90?

When relationships of this kind are true between pairs of fractions the fractions have equal values. We call these "equivalent fractions."

How Many? How Much?

Have you ever thought of how counting and measuring are alike and how they are different?

Do we use numbers when we count? Do we use numbers when we measure? That's right, we use numbers in both cases.

When we use numbers with measurement do we get different information than when we use numbers for counting separate objects? Let's see.

Imagine that you say, "We buy 2 bottles of milk a day at our house." Your friend says, "We buy one bottle of milk a day. You must buy twice as much milk as we do." Is your friend right?

Your friend may be right or he may be wrong. He made his judgment without enough information, and we don't have enough information to say whether he is right or wrong. What else do we need to know? That's right, the **measure** of the amount of milk each family buys. If your family buys 2 one-quart bottles of milk and your friend's family buys 1 two-quart bottle of milk, then both families buy the same amount of milk, don't they?

Counting the bottles of milk gave us the number of bottles, but did it give us the amount of milk? No. We had to know the measures in order to know how much milk each family bought daily.

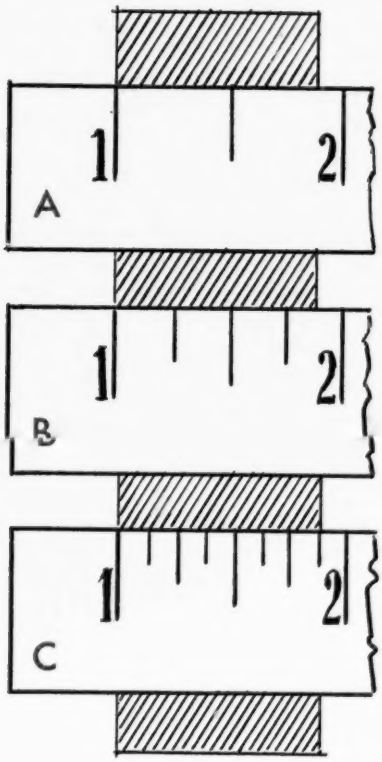
Try to answer the following questions. Then read **Checking Your Thinking** on page 39.

1. Bill has two balls of kite string. Sally has one ball of kite string. What information do you need before you can tell who has more kite string?
2. One store advertises sugar cookies at 35 cents a dozen. Another store advertises the same kind of cookies at 40 cents a pound. What do you need to know before you decide if one is a better buy than the other?

If we count separate objects correctly, we know exactly how many of them we have. Even when we measure an object very carefully, we never know exactly how much of it we have. We say that measures are **approximate** rather than **exact.**

The idea that measures are approximate seems to be difficult for people to understand. Perhaps some examples will help you begin to understand this idea.

Suppose that you are measuring a narrow board. Ruler A is marked in half-inches. What is the width of the board? Did you say 1 inch? That is correct. It is the best measure you can get with a unit of $\frac{1}{2}$ inch. Ruler B is marked in quarter inches. What is the width of the board? Did you say $\frac{3}{4}$ inch?

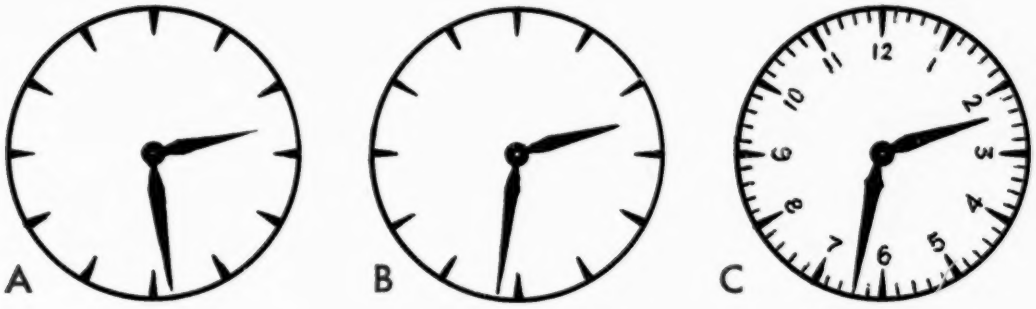


Ruler C is marked in eighth inches. What is the width of the board? Did you say $\frac{7}{8}$ inch? That's correct.

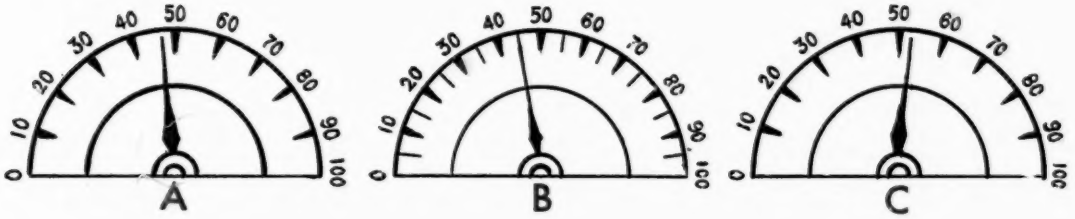
Do you see that when the unit of measure is smaller, you are able to get a measure which is closer to the width of the board? Can you imagine that even with the smallest possible unit the measure would not be exact in the same way that a counted number is exact?

Mathematicians say that the smaller the unit of measure the more precise the measurement will be. Through the discoveries of modern science we are able to make more precise measurements than ever before. In science, government, and industry we are beginning to measure many things electronically, and these measurements are much more accurate than those we have been able to make in the past. Light waves, sound waves, radio waves, and air flow will be the yardsticks of the future.

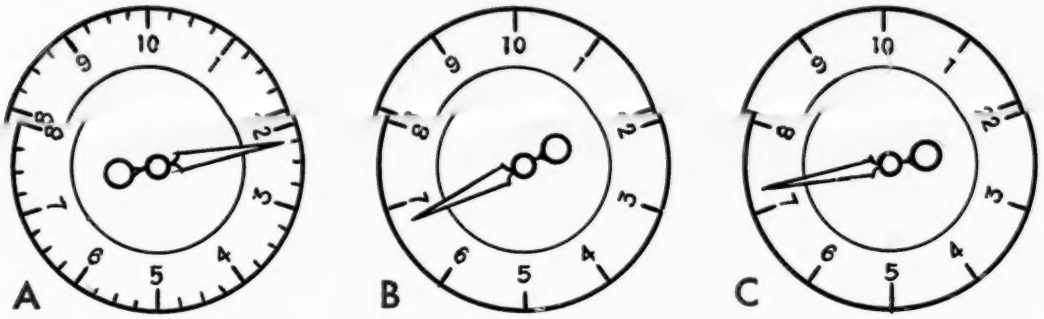
How Do You Read These Measures?



1. Approximately what time is it by each clock?



2. What is the approximate speed of each car?



3. Approximately what weight is shown on each scale?

CHECKING YOUR THINKING

1. The measure of the string in each ball.
2. The weight of the dozen cookies, or the number of cookies that weigh a pound.

Reading Measures

1. A. 2:30 B. 2:30 C. 2:32
   Do we often read time to the minute, as 2:32?
2. A. 50 mi. B. 45 mi. C. 50 mi.
3. A. $2\frac{1}{4}$ lb. B. 7 lb. C. 7 lb.

Units for Buying and Selling

Many measures are used for buying and selling things. Certain measures are usually used for certain things. For example, we buy cloth by the yard and sugar by the pound. See if you know what units would be used for buying and selling the following things.

1. gasoline
2. ribbon
3. potatoes
4. butter and margarine
5. bread
6. eggs
7. milk
8. nails
9. candy
10. apples
11. a board
12. ice cream

Copy the list above. Beside each item write the name of the unit by which you think each is sold. Then check your thinking on page 41.

Buying the "Giant Economy" Size

Bob and Betty often help with the family shopping. Do you ever help your family? If so, you may have noticed that some products are packaged in "giant economy" sizes. Many advertisements suggest that it is possible to save money by buying these large size packages.

Use the examples below to find out which size package is the better buy. You can estimate or calculate. Skip examples you can't figure out. Check your thinking on page 42.

1. Detergent A
   a. 23 lb. package costs $4.83
   b. 2 lb. package costs $ .52
2. Soap W
   a. 6 lb. package costs $1.56
   b. 2 lb. package costs $.56
3. Which package is the most economical?
   Study these prices for potato salad.
   a. 1 lb. (in cardboard carton) $.29
   b. 2 lb. (in plastic carton) $.63
   The potato salad in both containers is the same kind. Which size would you buy? Why?

Choose an item your family buys. Answer these questions about your shopping habits.

a. Which size of this item is best suited for your family's needs?
b. Do you save money when you buy this size?
c. Is this size the easiest to handle?
d. Do you have storage space for the large package?
e. Will any of this item be wasted or get stale if you buy the largest size?

CHECKING YOUR THINKING

Units for Buying and Selling

1.	gasoline	gallon
2.	ribbon	yard or bolt
3.	potatoes	pound
4.	butter and margarine	pound
5.	bread	loaf The weight is usually printed on the wrapper.
6.	eggs	dozen
7.	milk	pint, quart, and gallon
8.	nails	pound
9.	candy	ounce, pound, or piece
10.	apples	pound
11.	a board	board feet
12.	ice cream	pint, quart, or gallon

Do you like to make collections?

You may want to collect cans, bottles and boxes and see how the units of measure are used on the labels. You may find some which use both the English and metric units.

Do you like to find words in the dictionary?

Look up **gross, ream** and **bolt.**

Buying the "Giant Economy" Size

1. Figure the cost per pound.

   Divide the amount of money by the number of pounds to get the cost per pound.

   a. $23\overline{)\$4.83}$ — 21 cents per pound in large package

   $$\begin{array}{r} \$\ .21 \\ 23\overline{)\$4.83} \\ 46 \\ \hline 23 \\ 23 \\ \hline \end{array}$$

   b. $2\overline{)\$\ .52}$ — 26 cents per pound in small packages

   $$\begin{array}{r} \$\ .26 \\ 2\overline{)\$\ .52} \\ 4 \\ \hline 12 \\ 12 \\ \hline \end{array}$$

Buying the large package saves 5 cents per pound. 5 cents saved per pound on 23 pounds would be $1.15 saved.

2. a. $6\overline{)\$1.56}$ — 26 cents per pound in 6 lb. package

   $$\begin{array}{r} \$\ .26 \\ 6\overline{)\$1.56} \\ 12 \\ \hline 36 \\ 36 \\ \hline \end{array}$$

   b. $2\overline{)\$\ .56}$ — 28 cents per pound in 2 lb. package

   $$\begin{array}{r} \$\ .28 \\ 2\overline{)\$\ .56} \\ 4 \\ \hline 16 \\ 16 \\ \hline \end{array}$$

Buying the larger size saves 2 cents per pound. 2 cents saved per pound on 6 pounds would be 12 cents saved.

3. a. $.29 per pound

   b. $2\overline{)\$\ .63}$ — $2 \times \$.29 = \$.58$; $\$.63 - \$.58 = \$.05$

   $$\begin{array}{r} \$\ .31\frac{1}{2} \\ 2\overline{)\$\ .63} \\ 6 \\ \hline 3 \\ 2 \\ \hline 1 \end{array}$$

The 2 lb. package costs more per pound. You might buy it if you wanted the plastic container. It is costing you a nickel.

Adding and Subtracting Units of Measure

Do you know how to add and subtract units of measure? Can you find Bob's mistakes? He asked Betty to help him correct his arithmetic paper. Copy each example, rework it, and find Bob's mistakes. Then check your thinking on page 44.

1.
$$\begin{array}{rr} 1 \text{ gal.} & 3 \text{ qt.} \\ +\,2 \text{ gal.} & 2 \text{ qt.} \\ \hline 3 \text{ gal.} & 5 \text{ qt.} \\ =5 \text{ gal.} & 1 \text{ qt.} \end{array}$$

2. 3 lb. 2 oz. + 6 oz. =
9 lb. 2 oz.

3.
$$\begin{array}{rr} 3 \text{ yd.} & 2 \text{ ft.} \\ +\,4 \text{ yd.} & 1 \text{ ft.} \\ \hline 7 \text{ yd.} & 3 \text{ ft.} \end{array}$$

4.
$$\begin{array}{rr} 2 \text{ hr.} & 20 \text{ min.} \\ -\,1 \text{ hr.} & 30 \text{ min.} \\ \hline 1 \text{ hr.} & 10 \text{ min.} \end{array}$$

5.
$$\begin{array}{rr} 4 & 12 \\ 5\!\!\not\!\!\text{ ft.} & 2\!\!\not\!\!\text{ in.} \\ -3 \text{ ft.} & 4 \text{ in.} \\ \hline 1 \text{ ft.} & 8 \text{ in.} \end{array}$$

6.
$$\begin{array}{rr} 3 \text{ qt.} & 1 \text{ pt.} \\ -\,1 \text{ qt.} & 1\frac{1}{2} \text{ pt.} \\ \hline 2 \text{ qt.} & \frac{1}{2} \text{ pt.} \end{array}$$

In order to add and subtract units of measure it is necessary to know the equivalent units. If you are not sure about equivalent units, you may want to study the tables. Tables for Weight, Linear, and Time units are on pages 10, 12, and 26.

Units of Liquid Measure

2 cups	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)
3 teaspoons	=	1 tablespoon
16 tablespoons	=	1 cup
16 fluid ounces	=	1 pint

Units of Dry Measure

2 pints	=	1 quart (qt.)
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)

CHECKING YOUR THINKING

Correcting Bob's Work

1. Do you think Bob may believe that 2 quarts = 1 gallon? He changed 5 quarts to 2 gallons 1 quart.
   Did you work the example correctly?

$$\begin{array}{rr} 1 \text{ gal} & 3 \text{ qt.} \\ +\,2 \text{ gal.} & 2 \text{ qt.} \\ \hline 3 \text{ gal.} & 5 \text{ qt.} \end{array} = 4 \text{ gal. } 1 \text{ qt.}$$

2. Do you think Bob got his answer by adding 6 ounces to 3 pounds? He may not know that we must add like units.

   3 lb. 2 oz. + 6 oz. = 3 lb. 8 oz.

3. Did Bob forget to complete the example by changing 3 feet to 1 yard? If there are enough of the smaller units to equal 1 of the larger units, we change to the larger unit.

$$\begin{array}{rr} 3 \text{ yd.} & 2 \text{ ft.} \\ +4 \text{ yd.} & 1 \text{ ft.} \\ \hline 7 \text{ yd.} & 3 \text{ ft.} \end{array} = 8 \text{ yd.}$$

4. Could Bob have subtracted 20 from 30 rather than subtracting 30 from 80? Perhaps he forgot that he could change 1 of the hours to 60 minutes to make the subtraction possible.

$$\begin{array}{rr} 1 \text{ hr.} & 80 \text{ min.} \\ \not{2} \text{ hr.} & \not{2}0 \text{ min.} \\ -1 \text{ hr.} & 30 \text{ min.} \\ \hline & 50 \text{ min.} \end{array}$$

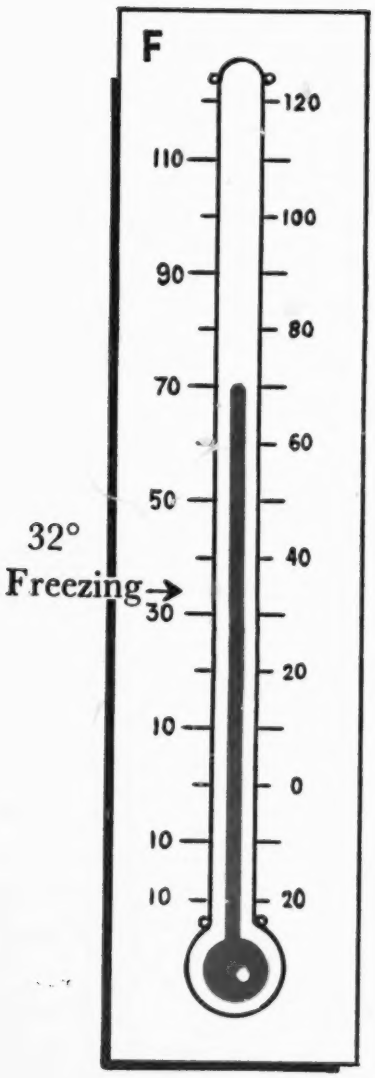
5. Did Bob forget that 1 foot equals 12 inches? He seems to have added 10 to 2 to get 12. Or perhaps he changed 1 foot to 12 inches but forgot to add the 2 inches.

$$\begin{array}{rr} 4 & 14 \\ \not{5} \text{ ft.} & \not{2} \text{ in.} \\ -3 \text{ ft.} & 4 \text{ in.} \\ \hline 1 \text{ ft.} & 10 \text{ in.} \end{array}$$

6. Did Bob understand how to subtract the fraction?

$$\begin{array}{rr} 2 & 3 \\ \not{3} \text{ qt.} & \not{1} \text{ pt.} \\ -1 \text{ qt.} & 1\frac{1}{2} \text{ pt.} \\ \hline 1 \text{ qt.} & 1\frac{1}{2} \text{ pt.} \end{array}$$

How Hot? How Cold?



"Come on, Betty," said Bob, "play ball with me."

"No, thank you," answered Betty. "It's too hot today."

What did Betty mean? How hot is a hot day? Have you ever checked a thermometer to see just what you mean when you say that a day is "hot"?

Thermometers measure how warm or how cold things are. There are many kinds of thermometers. One kind is made with a liquid in a bulb. A very thin tube runs up from the top of the bulb. The liquid in the bulb has no place to go but upward in the tube when it expands. Marks are placed at even intervals beside the tube. The mark at the height of the liquid measures how warm or how cold it is.

The unit for measuring temperature is the degree. The symbol for degree is a little circle. 72° is read 72 degrees.

Can you read a Fahrenheit thermometer like the one at the left?

How many marks are there from 0 to 10? 10 to 20? How many degrees does each line mark? That's right, 2 degrees.

Find the zero point on this thermometer. What point is the freezing point? Is it above or below 0°?

On a Fahrenheit thermometer, the freezing point of water is 32° above zero. Do you think, then, that 0° is colder than freezing? That's right, 0° is very cold.

What is the warmest temperature that could be read from our thermometer? Yes, the highest point shown on the measuring scale is 120. This thermometer is intended to measure temperatures of the air in a room or in the shade out-of-doors. The temperature in these places seldom goes above 120°, so a longer measuring scale is not needed.

Water boils at 212° Fahrenheit at sea level. Do you think that a cooking thermometer would be different from the thermometer in our picture? How? Why?

Temperatures Below Zero

Have you ever wondered about the temperatures at the North and South Poles? Do pictures taken there show a great deal of ice and snow? Would you guess that the temperatures are very, very low?

Water freezes at 32° Fahrenheit. When the temperature falls 5° below zero, how much below freezing is it? Use the thermometer pictured on page 45 to see. Did you get 37° by moving from 32° to 0° and then continuing 5 more degrees below 0°? Does $32 + 5 = 37$?

How many degrees below freezing would a temperature of 10 degrees below zero be? 20° below zero? That's right, 10° below zero is 42° below freezing, and 20° below zero is 52° below freezing. Could a temperature lower than 20° below zero be read from the thermometer pictured on page 45? Why not?

Do you think that temperatures more than 20° below zero are usual? That's right, they are unusual, but they do happen around the North and South Poles.

Temperatures below zero degrees are marked —10, —20, and so on to show that the numbers mark points in the direction below zero. We read —8° **minus 8 degrees.**

See if you can figure out how much below the freezing point of water the following temperatures are. Remember, we are using a Fahrenheit thermometer, as we did in all our discussion above. Check your thinking on page 47.

1. —125° Fahrenheit, lowest recorded temperature near the South Pole.
2. —300° Fahrenheit, temperature of liquid oxygen (**LOX**) used for rocket fuel.
3. —243° Fahrenheit, temperature scientists believe to be that of the moon's surface at night.
4. —487° Fahrenheit, temperature of liquid hydrogen used for experimental rocket fuel.
5. —518° Fahrenheit, temperature of liquid helium used in research.

CHECKING YOUR THINKING

1. $32 + 125 = 157$. **—157°** record low at South Pole.
2. $32 + 300 = 332$. **—332°** temperature of LOX.
3. $32 + 243 = 275$. **—275°** moon's surface at night.
4. $32 + 487 = 519$. **—519°** temperature of liquid hydrogen.
5. $32 + 518 = 550$. **—550°** temperature of liquid helium.

All answers show how much below the freezing point of water these cold temperatures are.

Very High Temperatures

Water boils at 212° Fahrenheit at sea level. How many degrees above the boiling point of water is 245°, the cooking temperature of caramel candy? Use the candy thermometer at the left to figure it out. Did you get 33°? You found the difference, didn't you. You could subtract, $245 - 212 = 33$. Or, you could think $212 +$ what number $= 245$. $212 + 33 = 245$. In either case the difference is 33. The cooking temperature of caramel candy is 33° above the boiling point of water.

See if you can figure out how many degrees Fahrenheit above the boiling point of water the following temperatures are.

1. 310°, cooking temperature of glazed fruit.
2. 1000°, furnace for making steel.

3. 2000°, molten lava at Hawaiian volcanic eruption in January, 1960.
4. 2400°, ceramic kiln firing pottery.
5. 7100°, combustion temperature of rocket fuel JP4.

Check your thinking below.

CHECKING YOUR THINKING

Very High Temperatures

1.	310 — 212 = 98	**98°**
2.	1000 — 212 = 788	**788°**
3.	2000 — 212 = 1788	**1788°**
4.	2400 — 212 = 2188	**2188°**
5.	7100 — 212 = 6888	**6888°**

Useful Knowledge About Temperature

Do you know these temperatures? Knowledge of them is often very useful to have. You might check to see if you know them.

1. Freezing point of water at sea level
2. Boiling point of water at sea level
3. Normal body temperature
4. Healthful room temperature
5. Safe refrigerator temperature
6. Safe temperature in a deep freeze box
7. Average home hot water temperature
8. Approximate temperature of a comfortable day
9. Moderate oven temperature

CHECKING YOUR THINKING

1. 32°
2. 212°
3. 98.6°
4. 68° to 72°
5. 36° to 40°
6. 0° to 2°
7. 140°
8. 70° to 80°
9. 325° to 375°

This Independent Learning Booklet is part of an experimental project to provide mathematics materials for able pupils and to develop a mathematics program adapted to individual differences. The project is based on several studies and exploratory programs carried on in San Diego County during the past several years, such as those sponsored by the San Diego Metropolitan Education Study Council and the Assistant Superintendents Group. Objectives of this phase of the project are to stimulate and widen pupil interest in various areas of mathematics; to develop deeper insights into mathematical principles, concepts, and relationships; to introduce aspects of modern mathematical thinking; and to develop motivation for able pupils to continue to study and explore mathematics and related fields.

The project is coordinated by the Department of Education, San Diego County, and directed by a steering committee selected from members of the San Diego Metropolitan Education Study Council. Support for the project is derived from funds provided under Title III of the National Defense Education Act of 1958 and matching funds from the participating school districts and the San Diego County Department of Education.

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Other Independent Learning Booklets have been prepared and will be announced as soon as they are ready for distribution.